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Poster

COLLABORATING TO DEVELOP ALGEBRAIC KNOWLEDGE FOR TEACHING

Meixia Ding, Xiaobao Li, Wei Chen
Temple University, USA, Widener University, USA, Nanjing Xiaozhuang Normal University, China

This poster reports NSF supported research that aims to seek Algebraic Knowledge for Teaching (AKT) based on the videotaped lessons of US and Chinese elementary expert teachers. Findings include AKT insights named "teaching through example-based problem solving" (TEPS) and teacher learning from cross-cultural videos.

OVERVIEW OF THE STUDY

Project and participants

This NSF supported research (2014-2020) aims to identify *Algebraic Knowledge for Teaching* (AKT) based on US and Chinese expert teachers' lessons. (https://www.nsf.gov/awardsearch/showAward?AWD_ID=1350068; https://sites.temple.edu/nsfcareerakt/). A total of 17 US and 17 Chinese teachers in grades 1-4 participated in the project. Each taught four lessons on either inverse relations or the basic properties of operations.

Procedures and Data analysis

To identify AKT, we analyzed, both quantitatively and qualitatively, 136 video-taped lessons and corresponding teacher interviews in terms of teachers' use of worked examples, representations, and deep questions (Pashler et al., 2007). In addition, we annotated 25 typical US and Chinese video clips and shared them with all participants through an online video forum and onsite summer workshops. Teachers' comments on videos were studied to understand what teachers may have learned from the cross-cultural videos.

Results

Identified AKT Insights: TEPS. Our video analysis of cross-cultural lessons shed light on AKT insights regarding how to effectively teach early algebra. We have reported these findings in several journal publications (e.g., Barnett & Ding, 2019; Chen & Ding, 2018; Ding, 2018; Ding, Chen, Hassler, 2019; Ding, Hassler, & Li., 2020). Together, these AKT insights suggest a unique instructional approach named, *teaching through example-based problem solving* (TEPS), which is documented in a recent book (Ding, 2021). This approach emphasizes engaging students in the process of working out an example task through pertinent representation uses and deep questioning (see Figure 1).

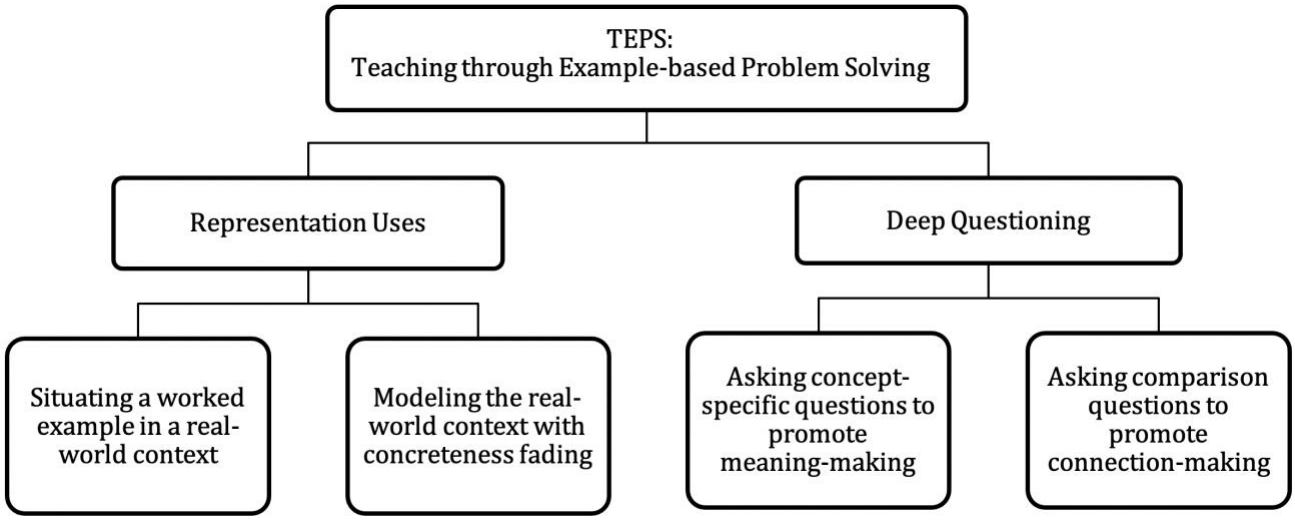

Figure 1. A summary of the key components of TEPS

As seen from Figure 1, with regard to representation uses, a teacher may situate a worked example in a real-world context (e.g., word problem), which can be modeled through "concreteness fading." For deep questioning, a teacher may ask concept-specific questions to promote meaning-making and comparison questions to promote connection-making. TEPS has been illustrated through selected Chinese and U.S. lesson episodes on two fundamental mathematical ideas, inverse relations and the basic properties of operations (see the detailed episodes in Ding, 2021).

Teacher Learning from Cross-cultural Videos. We shared our annotated 25 cross-cultural video clips with participating teachers in both countries through both online platforms (followed with 3-day summer workshops). The video comments (n=233) indicate that teachers in both countries paid greater attention to their international peers' videos (see Figure 2).

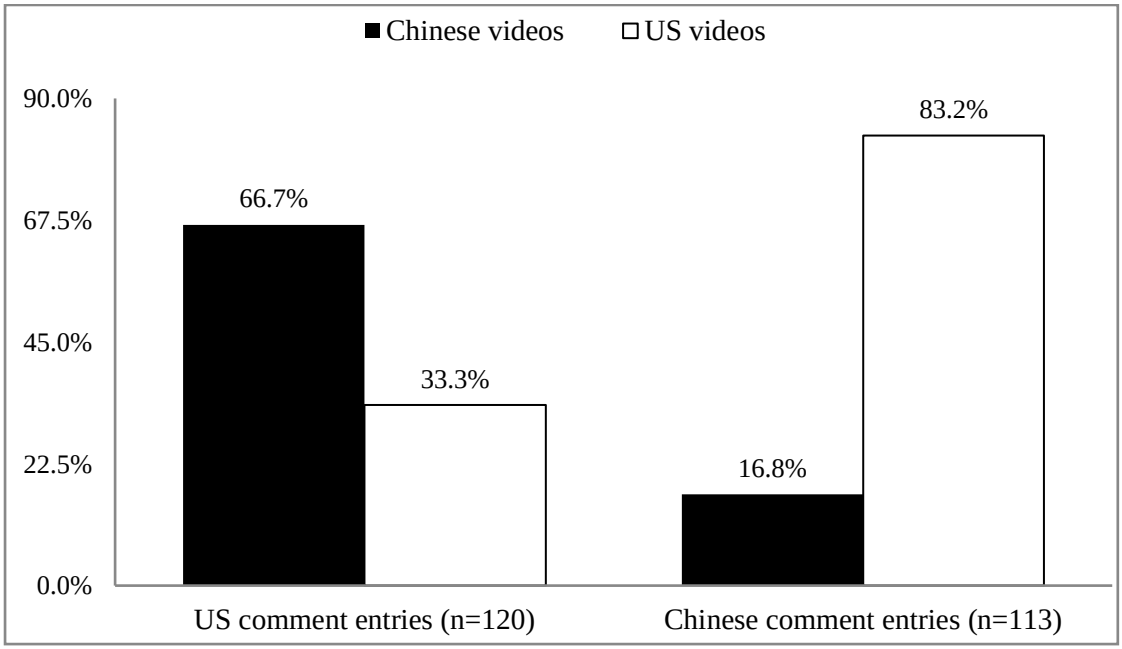

Figure 2. Teachers' comment distribution and the sources of comments on the videos.

We further segmented the video comments into 643 noticing codes. It was found that teachers in both countries commented more on "teaching" (59.2% of US codes, 65.9% of Chinese codes) than the other domains (mathematics, children, and other). In particular, US teachers were impressed by Chinese lessons' situating the worked examples in real-world situations, the concreteness fading approach, and deep questioning. On the other hand, Chinese teachers were impressed by US classrooms' relaxed atmosphere and the method of "combining numbers and diagram" indicated by the array model. Teachers at both sides made deep reflections and expressed strong desire to transfer their learning into their own classrooms.

Contribution and Future Research

Our cross-cultural findings contribute a balanced view to the research on worked examples and problem solving, which has been debated for decades. Findings also contribute to improving early algebra teaching and mathematics education reform in both US and China. Future studies should explore how teachers may implement what they've learned from international peers and how these cross-cultural video resources may make a broader impact on more elementary school teachers.

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