CONTEXTUALIZED ESTIMATION TASK SEQUENCE TO PROMOTE FLEXIBILITY IN PROBLEM SOLVING

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THEORETICAL FRAMEWORK AND PURPOSES

Using tasks in real-world contexts promotes meaningful learning in mathematics. In this study we use a subset of Fermi problems (Ärlebäck, 2009), those problems that require estimating a large number of elements in a real delimited area, such as knowing the number of people that fit in a public square.

Elia et al. (2009) consider strategy flexibility as the behavior of switching strategies during the solution of a problem (intra-task strategy flexibility) or between problems (**inter-task flexibility**). According to CCSSI (2010) flexibility in problem solving is related with mathematical competence. Nevertheless, teachers have still some difficulties to develop flexible reasoning when solving mathematical tasks. This leads us to consider whether pre-service primary school teachers know and use different strategies to solve Fermi problems. Actually, teachers should experience problem solving from the problem solver's perspective before adequately teaching it. In this sense, it is important to design task sequences that promote flexibility in problem solving. In Ferrando, Segura & Pla-Castells (2020) we identified a significant relationship between the characteristics of the real context of an estimation problem and the resolutions chosen by prospective teachers; and then we validated the design of a sequence of four contextualized estimation tasks (Ferrando & Segura, 2020).

These results led us to formulate the present research questions regarding inter-task and intra-task flexibility:

R1: Does such a sequence promote inter-task flexibility among prospective teachers?

R2: Is there a relationship between inter-task and intra-task flexibility when dealing with the tasks of this sequence?

METHODOLOGY

For the design of the sequence of task, we have considered 3 variables: the task structure and format variables were fixed, whereas values of context variable (size of the elements and of the region; arrangement of the elements) were identified and used to construct the sequence of four problems taking different values.

The experience was carried out during 2017/18 and 2018/19.
It involved N=224 prospective primary school teachers that worked individually for 45 minutes on the sequence of tasks.

Statement	Context variables values
P1- People. How many students can stand on the faculty porch when it rains?	Element size: medium Arrangement: irregular Region size: medium
P2- Tiles. How many tiles that are there between the education faculty building and the gym?	Element size: medium Arrangement: regular Region size: big
P3- Grass. How many blades of grass are there in this space?	Element size: small Arrangement: irregular Region size: medium
P4- Cars. How many cars can fit in the faculty parking?	Element size: bigArrangement: regular Region size: big

The qualitative analysis of the productions led us to categorize them into four types of resolution combining the emergent model and the strategy: *counting*; *linearization* (productions that distribute the elements by rows); *base unit* (productions based on the procedure of dividing the total area by the area of an element taken as a unit); *density* (productions based on the procedure of multiplying the total area by an estimated density). Those productions that did not provide enough detail to obtain an estimate were categorized as *incomplete*.

For the analysis of **inter-task flexibility** we excludes the productions of the 14 prospective teachers who had not been able to solve more than one, with the rest (N=210) the following scale was established: those who merely proposed the same type of resolution in two or more tasks were categorized as *not flexible*; those who proposed two different type of resolution were categorized as *flexible*; and finally, those who proposed three or more different resolutions were categorized as *very flexible*.

For the analysis of **intra-task flexibility**, we asked part of the solvers (N=110) to propose an alternative resolution in task P4 in order to analyze intra-task flexibility: solvers who were able to propose two different resolutions were categorized as *intra-task flexible*.

RESULTS & DISCUSSION

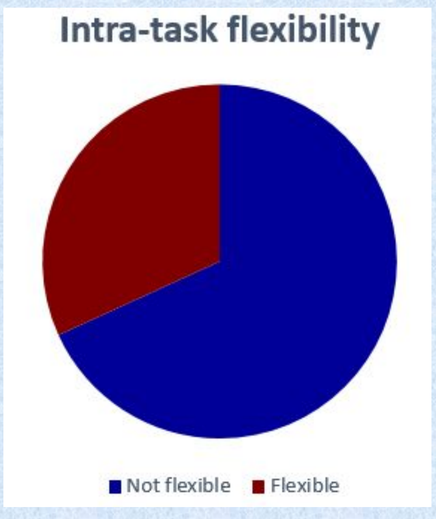
Inter-task flexibility

From this analysis it is deduced that the sequence of tasks designed promotes that a majority of prospective teachers propose two or more different resolutions when changing problems. Indeed, even with tasks as similar as those in this sequence, the variation of the context variables influences prospective teachers to change the type of resolution.

	Not Flexible	Flexible	Very Flexible
Abs. Freq.	49	**92**	69
Rel. Freq.	23,3%	**43,8%**	32,9%

Intra-task flexibility

Only 32% of the prospective teachers (35 in total) are categorized as intra-task flexible.



Relation between inter-task flexibility and intra-task flexibility

Of those 35 solvers, 88% had been categorized as flexible or very flexible inter-task in the sequence. However, the Chi-Square test of independence indicates **that there is no significant relationship between inter-task and intra-task flexibility** (Chi-Square (2, N=110) =2.7973, p=.24). The proportion of intra-task flexible prospective teachers is low; however, this is not sufficient to draw reliable conclusions, because difficulties in proposing an alternative resolution may be related, for example, to the context variables of the task P4. We must study whether context variables such as the size or arrangement of the elements are related to the difficulty of choosing an alternative resolution to the problem.

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