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POSTER TEMPLATE

DOES THE SQUARE ROOT OF A PRIME NUMBER IS IRRATIONAL?

−A CASE RESEARCH ON PROBLEM POSING AND SOLVING

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From the perspective of problem posing and solving, we give a complete case which can be implemented in mathematics curriculum in high school. We start on an initial problem: prove the square root of 2 is irrational. Then we pose a further problem that the square root of a prime number is also irrational. The concerning knowledge of solutions is elementary for high school students. This case of problem posing and problem solving can helps high school students to strengthen their competency of logic inference in mathematics to some extent, containing deduction, induction and analogy.

INTRODUCTION AND MAIN RESULTS

The aim of this research is try to give a classical example of problem posing and problem solving for the high school students that how to pose and prove the square root of one prime number is irrational by using geometry method. Pythagoras gave the classical proof on $\sqrt{2}$ is irrational by using algebraic method (Hardy, 1992). Figure 1 shows the chain of problems posing on "what is square root of 2 ?".

Prove $\sqrt{2}$ is irrational by geometry method.

Proof. As is shown in Figure 2, in the isosceles right triangle ΔABC , $AC = BC = q$, $AB = p$, $2q^2 = p^2$. Suppose that $\sqrt{2}$ is rational, there are exist two integers p and q , where p and q have no common factors, such that $\sqrt{2} = p/q$. Setting an arc of radius q intersecting AB with M , then setting a vertical line MN of AB , intersecting BC with N . Then $AM = q$, $BM = p - q$, $CN = p - q$, $BN = q - (p - q) = 2q - p$. Therefore $\sqrt{2} = p/q = (2q - p)/(p - q)$, but $2q < 2p$, hence $2q - p < p$ and $p - q < q$, it is a contradiction with our assertion that p and q have no common factors. Remark: This method refined and generalized the arguments by Apostol (Apostol, 2000). We can prove $\sqrt{3}$, $\sqrt{5}$ are irrational by the similar geometry method.

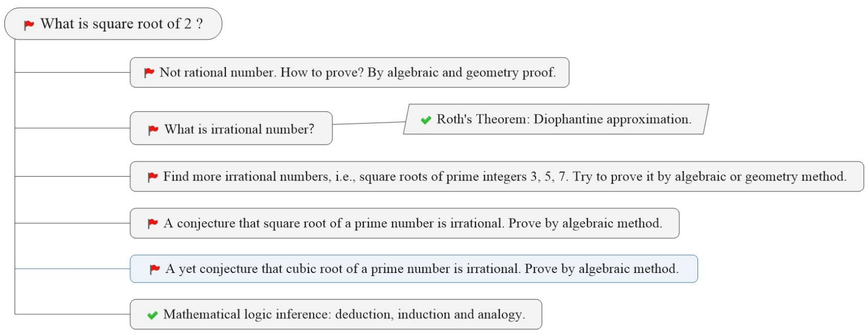

Figure 1: Chain of problems

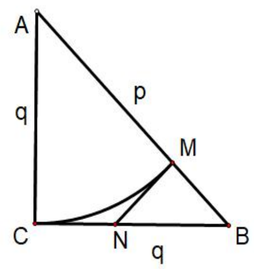

Figure 2: Right triangle ΔABC

References

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