The Last Decade of Proportional Reasoning A Systematic Review

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Abstract

The aim of this study is to investigate proportional reasoning articles published from 2008 to 2017 in the selected mathematics education journals by using content analysis method. The articles were investigated in terms of definition of the "proportional reasoning", method, source of data, sample/participants, subject (topic), and the results. It was seen that only half of the studies defined "proportional reasoning". The results of the articles those handled proportional reasoning from a broad perspective in terms of level of participants and subjects (topic) were categorized into three fundamental categories.

Introduction

Proportional reasoning, one of the most important topics of middle school mathematics, functions as a bridge in learning concepts in both higher mathematics and other knowledge areas. The importance of proportional reasoning and its long and complicated development process have made it subject of many scientific researches up to now. Lamon (2007) emphasized that researchers have not made much progress in teaching and learning of proportional reasoning and related issues for more than the last decade, and that there are many unanswered questions about it. Particularly, in research areas that are described as complex, such as proportional reasoning, it is important and necessary to go back occasionally and to reveal the missing points of that area by evaluating the latest scientific advances (Lamon, 2007).

Methodology

The aim of this study was to investigate research articles published from 2008 to 2017 in the selected mathematics education journals given below;

- Educational Studies in Mathematics
- Journal for Research in Mathematics Education
- Journal of Mathematical Behavior
- Mathematical Thinking and Learning
- Zentralblatt für Didaktik der Mathematik (ZDM)

The research articles were investigated by using content analysis method within the contexts of definition of the "proportional reasoning", method, source of data, sample/participants, subject (topic), and the results.

The sample of the research which was consisting 49 articles was determined by the keywords "proportional reasoning", "proportionality" and "multiplicative reasoning".

Each article included in the sample was read in detail and notes were taken on the article in the context of research questions. A temporary coding scheme, which was created during reading and taking notes, was continuously updated and used for all articles.

Results

The definitions of Proportional Reasoning and the Related Terms

It was seen that that only half of the 49 studies included in the analysis made definitions of proportional thinking and related terms. The table below summarizes the related terms used, the terms defined and the content of definitions of the defined terms.

Related Terms Used	The Terms Defined	The Content of Definitions

• Multiplicative reasoning
• Proportionality
• Multiplicative structures
• Linear reasoning
• Proportional relations
• Multiplicative relations
• Multiplicative concepts
• Linearity
	• Proportional reasoning
	• Multiplicative reasoning
	• Multiplicative concepts
	• Proportional relations
	• Multiplicative structures
		• The definition of proportional reasoning was made based on;
		  • Multiplicative comparison
		  • Mathematical thinking (additive versus multiplicative)
		  • The area studied (statistics)
		• The definition of multiplicative reasoning was made based on;
		  • Multiplicative relations
		  • Multiplicative schema
		  • Conceptual fields

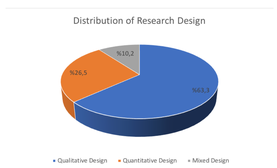
As stated before, in half of the 49 studies included in the study, the terms proportional reasoning and the related terms were mentioned, but it was seen that no exact definition was made.

Instead, in these articles the issues given below were mentioned;

- The importance of proportional reasoning as both a topic and a skill
- The difficulty and necessity of proportional reasoning
- Being a basic tool for interpreting real life phenomenon
- Underlying many mathematical ideas such fractions, ratios and linear functions

Research Designs

It was seen that out of 49 studies examined, 31 were designed qualitatively, 13 were designed quantitatively and 5 used mixed research design.

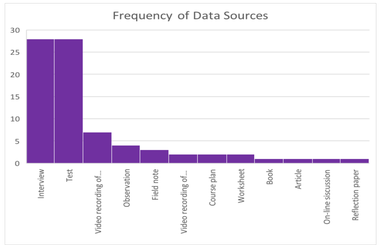


The percetages of research type for each research design is given in the following table.

Qualitative Design	Quantative Design	Mixed Design
Intervention (%55)	Survey (%85)	Intervention (%60)
Ascertaining (%35)	Intervention (%15)	Survey (%40)
Document Analysis (%55)		

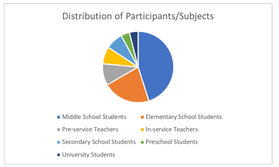
Data Collection Instruments

The articles investigated used at least one of the sources of data; meeting recordings, lesson plans, audio or video interview recording, worksheets, field notes, written answers and paper-pencil tests, observation notes, books, video recordings of practice sessions, articles, online discussions and reflective articles. As seen in the graph presented below, interviews and paper-pencil test were the most frequently used data collection instruments.



Participants/Subjects

When the levels of the participants/samples whose data were collected in the conducted studies were examined, it was seen that they were generally at middle school (23) and elementary school (11). In some studies, it was seen that working with students/teachers from both levels. Besides the middle and elementary school students, the studies also conducted with pre-service teachers (5), in-service teachers (4), secondary school students (4), preschool students (2) and university students (2).



Subjects (Topics)

It was seen that that the subjects (topics) of the research studies were handled from a broad perspective. First of all, proportional/multiplicative reasoning domain was investigated in the context of verbal problems. In these 11 studies missing value and comparison problems, problems including proportional reasoning errors, real life problems, problems requiring multiplication, linear problems were used. Besides these, other subjects (topics) that proportional reasoning were investigated were similarity (3), pattern generalization and algebra (5), ratio, proportion and enlargement (3), data literacy (1), fractions (5), percent (2), trigonometry (1), probability (2), chemistry, linear functions and equations (5), geometric shapes (2), Cartesian product (1), geometry (enlargement) (2), rational numbers (1), statistics (1), quadritic functions (1) , pie chart (1), measurement (4) and area measurement (1).

Results of the Articles

When it was seen that the results of the researches varied widely, the results of the 49 studies included in the sample were categorized into three fundamental categories. These results were related to;

- Processes of proportional/multiplicative reasoning (%47)
- Supporting proportional/multiplicative reasoning by appropriate environment/instruction/software/book (%39)
- Mastery and benefits of proportional/multiplicative reasoning (%14)

Conclusions and Recommendations

The difference between definitions of proportional reasoning and the related terms shows that there is no a common consensus among mathematics education researchers. In parallel with the fact that the majority of the studies examined used the qualitative research design, data were collected through interviews and paper pencil tests with the participants/subjects. In the studies conducted the participants/sample were predominantly at middle and elementary school. The variety of subject choices in the studies shows that ratio and proportion and related concepts are closely related to both other mathematical concepts and other fields of science. Since approximately half of the studies' results were related to support, improve and develop proportional/multiplicative reasoning processes, it can be interpreted that researchers attach great importance to this reasoning skill and to the development of this skill.

The recommendations are given as follows;

- In order to gain more knowledge in the area of proportional reasoning, a definition of this term that is adopted by everyone should be determined by the common efforts of mathematics education researchers.
- Both qualitative and quantitative longitudinal studies are needed to understand the development of proportional reasoning skills in more detail.
- It is strongly recommended that more proportional reasoning studies should be conducted with both in-service teachers to prepare professional development programs and undergraduate students to interpret reflection of proportional reasoning skills in advance mathematics courses.

References

Lamon, S. L. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In Frank K. L. (Ed.) Second handbook of research on mathematics teaching and learning (Vol 1, pp. 629-668). Charlotte, NC: Information Age Publishing.

Acknowledgements

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