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RESEARCH TO IMPROVE EDUCATION GUIDELINES FOR PROMOTING STUDENTS' UNDERSTANDING OF MATHEMATICAL FUNCTIONS

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In order to form a fundamental concept for 'relating two variables that quantify and capture the changes in actual phenomena' in the study of mathematical functions, we created a learning framework that consists of two stages: learning without the use of numerical values, and learning with the use of numerical values. We then conducted practical teaching sessions at the stage of learning without the use of numerical values. This was shown to have an effect on the formation of concepts for understanding functions, suggesting that there is potential to promote such understanding.

In the study of mathematical functions during primary and secondary schools (for students between 9 to 15 years), the basis for understanding consists of (I) Quantifying changes in a phenomenon, that is, deriving variables from the phenomenon, and (II) Establishing the relation between the two variables derived. According to the results of past studies, students' fundamental concepts of (I) and (II) have been found to be lacking. Research related to comprehension in mathematics education states that in order to develop abstract mathematical concepts, an initial phase of learning without the use of numerical values is integral, followed by a phase of learning with the use of numerical values (Dixon, Moore, et al. 1991, 1996). Accordingly, with the stated goal of assisting students to understand concepts of (I) and (II), we established an educational framework (Fig. 1). We developed teaching material with a focus on the stage of learning without the use of numerical values and conducted practical teaching sessions. Furthermore, the concepts of (I) and (II) at this stage are defined as being logically structured so that the following premises apply to the relationship between the two variables: 'As one variable increases, the other increases', and 'As one variable increases, the other decreases'.

The teaching material that we developed is used as students observe and record familiar phenomena, while studying concepts like how the length of a cone's shadow changes with the change in time (Fig.2). They first derive variables that correspond to variations over time, then study the relationship between the two variables that involve time passes.

For this purpose, we conducted practical teaching sessions with four primary school students, two from 9 to 10 years and two from 10 to 11 years.

From our surveys conducted during and after the session, we learned that it facilitated the formation of the students' fundamental concepts of (I) and (II) to some extent, thus providing a base for developing new education guidelines for the study of mathematical functions.

Learning without the use of numerical values		Learning with the use of numerical values
Attempts to develop the concepts of (I) and (II) by observing changes in a cone's shadow.	⇒	On the foundation of the concepts of (I) and (II), expressing such changes in the form of numbers, charts, and graphs, and thereby considering the relationship between the two variables.

Fig. 1 Educational Framework for the Formation of Concepts Regarding (I) and (II)

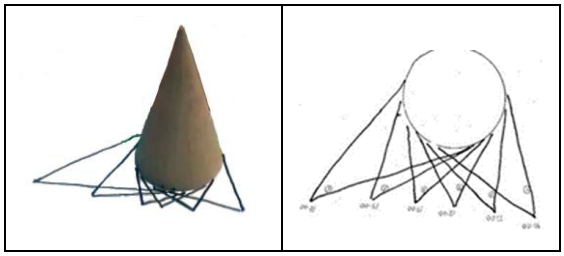

Fig.2 The recording of shadow changes (left), the shape of the recorded shadow (right)

References

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