

## WS1: Beyond Financial Literacy and Financial Mathematics: Conceptualizing Financial Numeracy

July 14, 21:30–23:00

Location: T419

**Organiser:** Annie Savard (McGill University, Canada); Alexandre Cavalcante (University of Toronto, Canada); Daniela Caprioara (Ovidius Universitatea din Constanta, Romania)

### Description:

This workshop aims to engage participants in an inquiry-based environment to conceptualize the role played by mathematics education in regard to financial literacy both in elementary and secondary schools. Our main goal is to introduce a conceptual framework that allows researchers and teachers to move beyond financial literacy and financial mathematics. The international efforts to incorporate financial literacy in schools have also penetrated the community of mathematics educators in several countries (Savard & Cavalcante, in press; Savard, Cavalcante, Turineck & Javaherpour, 2020; Caprioara, Savard & Cavalcante, 2019; Savard, 2018; Sawatzki, 2017; Bansilal, 2016), however the role played by mathematics in this topic has been undertheorized, leaving practitioners without proper support to integrate these concepts in mathematics classes.

The concept of financial numeracy refers to the intersection of mathematics and everyday practices in the realm of finance (Camiot & Jeanotte, 2016). In that sense, it comprises more than simple arithmetic operations in financial contexts (a role that has been proposed by scholars in the field of financial literacy). We believe that financial numeracy involves the wider scope of mathematical concepts, tools and procedures instead of referring to financial mathematics (which is a specific subfield of applied mathematics). In this workshop, participants will define their vision of the role played by mathematics educators in regard to the teaching of financial numeracy by exchanging their thoughts, categorizing tasks and by analysing some data.

### Planned Activities & Working Format & Responsible Person

- 21:30–21:45**     **Introduction + Developing a shared vision of financial numeracy**  
Participants will identify some links between financial literacy and mathematics education. They will answer questions about their motivation and understandings of financial numeracy.
- 21:45–22:05**     **Comparing and contrasting financial tasks**  
Participants will be presented with financial tasks that portray money in different ways. In groups (elementary and secondary schools), they will find similarities and differences between these tasks and identify the mathematics and financial literacy.
- 22:05–22:20**     **Moderated discussion of the findings**  
The presenters will moderate a discussion on the findings from participants' group discussions.
- 22:20–22:35**     **Presentation of a theoretical framework**  
The presenters will present their conceptual framework. They will also provide some readings for participants to be familiarized with the notion of financial numeracy.
- 22:35–22:50**     **Critical discussion on financial numeracy in schools**  
Participants will have the opportunity to share their concerns, challenges, perspectives and experiences regarding financial numeracy in schools around the world.
- 22:50–23:00**     **Conclusion**  
We aim to create a community centered around financial numeracy internationally. We will share resources and contact information for expanding these financial numeracy ideas.

## WS2: International Mathematics Festival: A Fun and Collaborative Event for Students to Discover “Why” and “What If”

July 14, 21:30–23:00

Location: T423

Organiser: Mark Saul (Open House Education Foundation, China); Cherry Pu (Open House Education Foundation, China); Rick Sommer (Stanford University, USA)

### Description:

A mathematics festival is an extra-curricular activity for K-12 students which provides them with advanced and attractive mathematics in a non-competitive situation. Rather than textbook exercises, a festival offers colorful puzzles, games, and hands-on activities to inspire students to think creatively and encourage collaborative problem-solving. Each activity comes with a wide range of problems on a given topic that students of different abilities can work on. Students can choose from among the activities freely, decide for themselves how long to work at each activity, and move to another table at their pace. The festival supports students' thought-process, helping them to discover mathematical patterns and relationships and to take joy in finding new insights. The length and depth of student engagement provides a useful evaluation of the activity.

We have originally planned to host a full-scale in-person mathematics festival at ICME-14. We have switched to an online version. At the beginning of the workshop, we will demonstrate a festival game and illustrates the progressive style of questions that students can explore. Then mathematics teachers and students will share their experiences. We will also conduct a live discussion with Rick Sommer, Hector Rosario and mathematics professionals who have extensive experience in K-12 mathematical enrichment programs.

### Planned Activities & Working Format & Responsible Person

21:30–21:40	<b>Welcome and introduction</b> Online/ Mark Saul and Cherry Pu
21:40–22:15	<b>Demo class of a festival game with international students joining online</b> Online/ Mark Saul and festival table facilitator
22:15–22:30	<b>Interview with students and teachers</b> Online and pre-recorded video / Mark Saul and Cherry Pu
22:30–22:45	<b>Panel discussion between Rick Sommer, Hector Rosario, Mark Saul and guest speakers on the K-12 mathematics enrichment program</b> Online/ Mark Saul and Cherry Pu
22:45–23:00	<b>Q&amp;A for conference participants</b> Online/ Mark Saul and Cherry Pu

## WS3: From the Power of Intuition to the Beauty of Abstraction

July 14, 21:30–23:00

Location: T519

Organiser: Damjan Kobal (University of Ljubljana, Slovenia)

### Description:

Contrary to the fact that mathematics many ideas, beauty and inspiration are hidden within simple and intuitive patterns, which are easily noticed and ‘intuitively understood’, mathematics is considered very abstract. Therefore, the motivation for mathematics teaching and learning should be intuitive and the beauty of abstraction will rise from there.

The main aim of the workshop is to challenge our sensibility for the importance of the intuitive in mathematics teaching and learning. Participants will be challenged by smartly chosen hands-on (and eyes-on) problems. Like some ‘graphic puzzles’, which are understood in seconds, but are often harder to formulate than to solve. Through examples participants will explore how understanding, motivation and challenge often lie within intuitive comprehension and how abstraction (especially on the primary level) only follows later. For better imagination of what will be happening at the workshop, participants are invited to explore interactive applets at <http://ko.fmf.uni-lj.si/ICME-14/> and at <https://www.geogebra.org/m/yknjxne>.

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Damjan Kobal has been a devoted teacher at high school and university level and a leading Slovenian educator for (high school) mathematics teachers. For many years he has been the leader of traditional seminars for mathematics teachers and frequent lecturer for teachers and students of all ages. He was a plenary speaker at the first international GeoGebra conference in Hagenberg, Austria, 2009 and has lectured on many international conferences and was hosted at many educational institutions from USA to India.

### **Planned Activities & Working Format & Responsible Person**

- 21:30–21:40**     **Introduction**  
Presentation – discussion / Kobal
- 21:40–22:50**     **Eyes-on problem solving**  
Dynamic exploration / Kobal
- 22:50–23:00**     **Sum it up and feed back**  
Discussion / Kobal

## **WS4: Learning with Virtual Manipulatives**

**July 14, 21:30–23:00**     **Location: T523**

**Organiser: Philipp Legner** (Mathigon, UK)

### **Description:**

Manipulatives have been used in mathematics classrooms for many decades: they can transform how students engage with mathematical ideas by making abstract relationships more visual, by teaching creativity and problem-solving skills, and by allowing students to explore and discover. Similarly, virtual manipulatives have greatly increased in popularity in recent years – especially for remote learning. They can mirror the effects of their physical counterparts, and even support more complex interactions that are not possible in the physical world.

In this workshop, we will explore a wide range of manipulatives: from the popular numbers bars, fractions bars and algebra tiles to more unusual ones like prime factor circles, multiplication grids, spinners, non-transitive dice, balance scales, exploding dots, polyominoes or tangram. We will share a brief summary of existing research into virtual manipulatives, as well as many different examples and activities how manipulatives can be used in the classroom to engage students in deep mathematical thought: with topics ranging from simple arithmetic to advanced number theory. We will also show participants how they can use some of the free tools we have developed at polypad.org to conduct research studies. Most importantly, we want to hear participants' thoughts, ideas and experiences in order to aggregate a library of teaching ideas for virtual manipulatives, come up with innovative, new ideas for manipulatives to build in the future, and understand which data we are collecting would be most useful for researchers, so that we can make it openly available.

### **Planned Activities & Working Format & Responsible Person**

- 21:30–21:40**     **Introduction and research summary**  
Lecture
- 21:40–22:00**     **Using manipulatives: existing websites, tools, activities and resources**  
Interactive lecture: participants can follow along on their own devices
- 22:00–22:10**     **Using Polypad to conduct research studies**  
Lecture
- 22:10–22:25**     **Participants share their experience and thoughts on using/researching manipulatives**  
Open discussion
- 22:25–22:45**     **Come up with specific ideas for classroom activities, new types of manipulatives or research projects using manipulatives**  
Break out into smaller groups
- 22:45–23:00**     **Student data: what is most interesting for researchers? What data formats to use?**  
Open discussion

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## WS5: Problem-based Learning: Enhancing Higher-order Thinking Skills in the Topic of Differentiation through STEM Approach

July 14, 21:30–23:00

Location: T116

**Organiser:** Sarveswary Velayutham (SMJK Chung Hwa Confucian, Penang, Malaysia); S.Kanageswari Suppiah Shanmugam (Universiti Utara Malaysia, Perlis, Malaysia); Tan Phei Ling (SMK(P) Methodist, Penang, Malaysia); Chia Hui Min (The Education University of Hong Kong, Hong Kong SAR, China)

### Description:

STEM have attracted the attention of researchers and educators from all over the world. There is increasing effort in promoting students' learning in STEM related subjects. However, study conducted by Nadirah Mohd. Nasir and her team (2013) found that students have difficulties in applying the concepts and basic problem-solving skills on differentiation related with science and engineering subjects. Similar findings were also reported by Wright (2014) and suggested that extra cognitive task and abilities were needed in solving algebraic word problem. As such developing higher order thinking skills in the topic of differentiation warrants attention. Hence, this workshop is aimed at adopting Problem-based learning through STEM for participants to experience hands-on in applying the concept of differentiation to enhance HOTS. The activities in this workshop are designed to facilitate students' HOTS through enquiry-based learning. With the use of GeoGebra, an open source software, participants will explore on the concept of tangent, perpendicular lines, minimum and maximum value, rate of change and small changes to construct models with low cost material and solve three problems involving concept of maximum and minimum value. Appropriate use of computer software could help to enhance students' learning in topic related to calculus (Bognar et al., 2018; Cekmez, 2020). Besides, Engineering the models promotes participants visualization, relational learning and application of STEM in real life and hence meaningful learning occurs from the series of STEM integrated activities. After the workshop, participants would be more confident in facilitating their students' inquiry learning in the topic of Differentiation and spur their interest in Mathematics.

### Planned Activities & Working Format & Responsible Person

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|--------------------|---|
| <b>21:30–21:40</b> | <b>Introduction to the workshop and Pre-workshop questionnaire</b><br>Colored sticky notes, discussion/ Chia, H. M. & Dr Sarves               |
| <b>21:40–22:00</b> | <b>Understanding the model (interactive discussion)</b><br>Power point/ Dr Kanages & Tan P. L   |
| <b>22:00–22:30</b> | <b>Case study 1 – Students perspective &amp; Case study 2 – Educators perspective</b><br>Power point, interactive/ Tan P. L. & Dr Sarves      |
| <b>22:30–22:35</b> | <b>Question and answer sessions</b><br>Discussions/ Dr Kanages  |
| <b>22:35–22:45</b> | <b>Post-workshop questionnaire &amp; making reflection from the sticky notes from slot 1</b><br>Colored sticky notes, discussion / Chia H. M. |

## WS6: The Felix Klein Project – Vignettes in Practice

July 14, 21:30–23:00

Location: T218

**Organiser:** Hans-Georg Weigand (University of Wuerzburg, Germany); Michelle Artigue (University of Paris, France); Christian Mercat (IREM de Lyon, France); Ferdinando Arzarello (University of Torino, Italy); Yuriko Baldin (Federal University of São Carlos, Brazil); Bill McCallum (University of Arizona, USA); Samuel Bengmark (University of Gothenburg, Sweden)

### Description:

The Klein Project aims to present contemporary mathematics for secondary school teachers. The idea of the project is to transfer the ideas of the legendary books of Felix Klein: “Elementary Mathematics from a Higher Standpoint”, written in the beginning of the 20th century, into the present. A collection of Klein Vignettes is found on the website (<http://blog.kleinproject.org>) in different languages. A Klein Vignette is a short article about a single mathematical topic. Vignettes are intended to give teachers a sense of

connectedness between the mathematics of the teachers' world and contemporary research and applications in the mathematical sciences.

**Aims and ideas:** Klein Vignettes are for teachers, but we also want to motivate them to bring ideas presented in the vignettes to the classroom. In some years of experience, we noticed that a) the ideas of the vignettes have to be supported by activities in the frame of professional development, and b) teachers had difficulties with the transfer of the Klein-ideas into the classroom, they had difficulties in creating adequate classroom materials.

This workshop pursues three aims:

- a) We want to give best practice examples how the idea of the vignettes could be integrated into the professional development of secondary school teachers;
- b) We want to motivate mathematicians to contribute to the Klein project with a new vignette;
- c) We want to motivate especially mathematics educators to think about Bridging-Vignettes which bridge the gap between the mathematics explained in a classical vignette and its use in the classroom.

### **Planned Activities & Working Format & Responsible Person**

- 21:30–21:40**     **Introduction to the ideas of the Klein Project**  
Presentation/ Hans-Georg Weigand.
- 21:40–22:00**     **Creating vignettes and exploiting them with teachers and students: The case of a vignette on entrelacs**  
Presentation of the vignette on Entrelacs, its story and associated resources. Practical work of design of entrelacs from some selected graphs. Michèle Artigue, Christian Mercat (France).
- 22:00–22:20**     **Working with Klein Vignettes as teaching strategies in actual classrooms – issues and possibilities. Material for professional development and designing innovating didactical sequences**  
Slide presentation / Report and interactive group discussion on given examples/ Yuriko Yamamoto Baldin (Brazil).
- 22:20–22:40**     **From a Klein Vignette to a concrete material for the classroom: the secret message game. Contribution from a work done with Italian teachers**  
Slides presentation/ using a software (PARI-GP) for illustrating the game/ short discussion with the ground / Ferdinando Arzarello (Italy).
- 22:40–23:00**     **Klein vignettes and problem-based instruction**  
We will give an example of how a Klein vignette can be adapted into materials for a workshop for teachers on problem-based instruction. Bill McCallum (USA).



## **WS7: Developing Quality Criteria for Creating and Choosing Mathematics Learning Videos**

**July 14, 21:30–23:00**     **Location: T316**

**Organiser:** Iresha Ratnayake (Technical University of Darmstadt, Germany); Eugenia Taranto (The University of Catania, Italy); Regina Bruder (Technical University of Darmstadt, Germany); Maria Flavia Mammana (The University of Catania, Italy)

### **Description:**

In this workshop, we share the results of a joint project between two universities in Germany and Italy. The project's aim is to develop quality criteria to create or to choose mathematical learning videos. There are many mathematical learning videos freely available on the internet and daily many videos are uploading on various platforms. However, there are many important factors that need to be considered in creating and choosing a learning video. Thus, in our project, we suggest some quality criteria that are crucial to accomplish the intention expressed above. We started with a catalogue developed under the CAKE project (Feldt-Caesar & Bruder, 2018). This catalogue was a general one including quality criteria for digital learning environments. Following this, the current project was designed to develop quality criteria for learning videos from a mathematical perspective (Ratnayake et al., 2020). The result of our collaboration has generated two catalogues: (i) quality criteria for creators and (ii) quality criteria for users (teachers). In particular, our catalogues paid attention to learning situations, expected prior knowledge, accuracy of the content, learner's expectations, pedagogical consideration and design and technical consideration. During this workshop, we share the two developed catalogues to stimulate discussion with

colleagues in the mathematics education research community about ways in which they might be refined and extended and contribute to building a shared understanding of creation and the use of high-quality mathematics learning videos.

### Planned Activities & Working Format & Responsible Person

- 21:30–21:40** **Watch learning videos and choose your favourite for your teaching intention - depending on a specific goal**  
The organizers show some videos (chosen from YouTube) created to accomplish some mathematical teaching intentions. The participants choose the best one that they would like to use in their teaching while noting down the reason/s for their choice.
- 21:40–21:50** **Presentation and use of the catalogue “Quality Criteria for Teachers as Users”**  
The organizers provide a link to the catalogue for users to the participants. The participants, individually, evaluate each video that they have watched using the catalogue which includes a 4-point Likert-style scale. They then, will share the evaluations in a Padlet and will start a discussion to choose the best one according to the results of the evaluations in small groups (depends on the number of participants).
- 22:50–22:05** **Group work and participants’ feedback to the catalogue for users**  
Based on this experience in the groups participants will then write down some suggestions in a Padlet, and/or critique, if any, about the catalogue for future use. They will write down these suggestions and will present at the discussion. Finally, the organizers conduct a discussion based on these suggestions.
- 22:05–22:25** **Watch learning videos from the designer’s point of view**  
The organizers show to the participants two videos created by teacher-students of each of the organizers’ universities.
- 22:25–22:35** **Presentation and the use of the catalogue “Quality Criteria for Video Creators”**  
The organizers provide a link to the catalogue for creators. The participants, individually, evaluate each video they watched using the catalogue which includes a 4-point Likert-style scale.
- 22:35–22:50** **Group work and participants’ feedback to the catalogue for creators**  
The organizers direct the participants to share their evaluations in a Padlet in groups. They will then discuss on participants’ suggestions, and/or critique, if any, about the videos and/or the catalogue in groups in breakout rooms. Finally, the organisers will conduct a discussion on these suggestions.

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## WS8: Rich Math Activities for a Primary School Class

**July 14, 21:30–23:00** **Location: T418**

**Organiser: Vilalta, A.** (Universitat Autònoma de Barcelona (UAB), Innovamat & eXplorium, Spain); **Morera, L.** (Universitat Autònoma de Barcelona (UAB), Innovamat & eXplorium, Spain); **Solar, H.** (Pontificia Universidad Católica de Chile (UC), Chile); **Rojas, F.** (Pontificia Universidad Católica de Chile (UC), Chile)

### Description:

Innovamat is a project that develops resources to teach and learn maths at school. In 2020/21 almost 700 Primary Schools are using our program, mostly in Spain but also in France, Chile or Ecuador. According to NCTM Principles and Standards, we define 4 big maths processes: “Problem-Solving”, “Reasoning and Proof”, “Connections” and “Communication and Representation”. Our activities always seek that children develop content and skills related to these processes, and this is why our teacher’s guides focus on class conversation and materials manipulation. In addition to that, Innovamat is making big efforts to train Primary School teachers in teaching rich maths from a process point of view.

Our didactic team is led by experts from Universitat Autònoma de Barcelona (UAB) and the ideas beneath the project are based on research. Specifically, our main sources are Morera’s doctoral thesis (directed by Fortuny, J. M., and Planas, N.), the Freudenthal Institute for Science and Mathematics Education, the Catalan official curriculum, USA Common Core and Mogens Niss’ research. At this workshop, we are going to introduce, perform and analyse two examples of activities from our project. Therefore, this

workshop might be especially interesting for Primary School teachers and any person who wants to know about rich activities and discuss them. We are going to explain two activities, ask attendants to take part in solving every challenge like if they were students, and concurrently we are going to discuss, as teachers, the maths didactics and learning opportunities beneath all of it.

### Planned Activities & Working Format & Responsible Person

- 21:30–21:45**     **Definition of the framework: maths processes in the Innovamat project**  
Computer and projector. / Exposition and group discussion. / Vilalta, A.
- 21:45–22:15**     **Activity 1: Geometry**  
Computer and projector, paper and three different coloured pencils. / Group discussion.  
/ Vilalta, A. & Morera L.
- 22:15–22:45**     **Activity 2: Productive thinking**  
Computer and projector, paper. / Group discussion. / Rojas, F. & Solar H.
- 22:45–23:00**     **Conclusions**  
Computer and projector, paper. / Group discussion. / Vilalta, A.

## WS9: Modelling Motion

**July 14, 21:30–23:00**                      **Location: T120**

**Organiser: Brian Doig** (Deakin University, Australia); **Susie Groves** (Deakin University, Australia); **John Cripps-Clark** (Deakin University, Australia)

### Description:

This workshop builds on the results of a research project, Modelling Motion: Developing Mathematics Concepts through STEM activities, which was funded by the Australian Association of Mathematics Teachers and the Australian Academy of Science. The activities were developed from the work of Galileo Galilei (1564-1642) who challenged the scientific wisdom of the Age that had been established by Aristotle.

In this workshop, attendees will perform activities, based upon the materials now available on the world-wide web, with a view to establishing the rôle of such activities as a part of a STEM programme that develops students' new mathematics, and not merely employs already known mathematics as a tool.

### Planned Activities & Working Format & Responsible Person

- 21:30–22:00**     **Session 1**  
This session will begin with some background to the work of Galileo and the reSolve Project. As many of the activities in Modelling Motion require measurement of speed in term of time and distance, it is necessary to investigate what speed means conceptually.  
A video of students investigating speed will be shown and the results discussed.  
Note that the use of streamer graphs enables the mathematics to explain the physics.  
Video with follow up discussion. / Brian Doig and John Cripps-Clark to lead the session.
- 22:00–22:30**     **Session 2**  
In this session we will employ Galileo's 'gravity diffuser' that allows us to measure how far a ball travels during successive time intervals. This was a crucial experiment that allowed Galileo to attack the Aristotelian precepts of motion. This activity builds on our experiences from Session One. The mathematics, again, is fundamental to explaining the physics. Finally, we will experiment with what happens to objects that are free-falling under gravity, and thus establish a mathematical relationship between distance fallen and time. Discussion will, of course, round off our activity.  
Video with follow up discussion. / Brian Doig and John Cripps-Clark to lead the session.
- 22:30–23:00**     **Session 3**  
In this final session we will investigate what happens when a motion is the result of two forces. While this is a simple experiment, the mathematics is not so simple for those of us with experience with a Cartesian plane.



A video of the experiment will be followed by a discussion that will recapitulate the set of activities viewed and discussed in all three sessions and also discuss some of the other activities from the Modelling Motion research.

Video with follow up discussion. / Brian Doig and John Cripps-Clark to lead the session.

## **WS10: Poly-universe & Lénárt Sphere: Manipulatives from Hungary**

**July 14, 21:30–23:00**

**Location: T124**

**Organiser:** Eleonóra Stettner (Hungarian University of Agriculture and Life Sciences, Hungary); Zsuzsa Dárdai (Poly-Universe Ltd, Hungary); István Lénárt (Eötvös Loránd University, Hungary); János Saxon Szász (Poly-Universe Ltd, Hungary); Réka Szász (Budapest Semesters in Mathematics Education, Hungary); Szabina Tóth (Szabó Lőrinc Bilingual Primary and Secondary School, Hungary)

### **Description:**

**Tools for Participants:** In order to enhance the experience, we recommend participants to have the following tools available during the workshop (they are not required): laptop or tablet, two oranges, coloured pens or markers that can mark the oranges, rubber bands, toothpicks, bottle caps.

Hungary has a strong tradition of using games and manipulatives to develop concept building and problem solving in mathematics. The workshop presents online adaptations of two educational tools: the Lénárt Sphere developed by István Lénárt, and the Poly-Universe set developed by János Saxon Szász. Both tools are used with 6-18 year-old students and in teacher training.

The aim of the Lénárt sphere is to explore analogies and differences between the plane and the sphere, which helps students understand how relative all axioms and theorems of science are, and also develop understanding of those who are different in their cultural or social background. The workshop will demonstrate how ball geometry can be taught online or in person using simple tools such as oranges and rubber bands.

The Poly-Universe (PUSE) is an educational tool that originates from art, and connects multiple subjects and mathematical topics through scale shifting symmetry and color combinations. In the workshop participants will have the opportunity to try out and discuss the e-learning platform of the tool, which is an online application for manipulating the set and solving tasks that involve both analytic and visual thinking.

### **Planned Activities & Working Format & Responsible Person**

<b>21:30–21:35</b>	<b>Introduction</b> Lecture / Szász
<b>21:35–22:15</b>	<b>Ball Geometry Activities</b> Interactive / Lénárt
<b>22:15–22:55</b>	<b>Poly-universe Activities</b> Interactive / Dárdai, Saxon, Stettner, Tóth
<b>22:55–23:00</b>	<b>Wrap-up</b> Discussion / Szász

## **WS11: Math for All: Professional Learning to Help Teachers Reach All Students in the Mathematics Classroom**

**July 14, 21:30–23:00**

**Location: T128**

**Organiser:** Babette Moeller (Education Development Center, USA); Matt McLeod (Education Development Center, USA)

### **Description:**

Persistent differences in mathematics performance between general and special education students underscore the need for improving teachers' preparation to better serve the needs of students with different strengths and needs. Math for All (MFA; Moeller et al., 2012; 2013) is an evidence-based, intensive, 50-hour professional learning program designed to help general and special education teachers

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in grades K–5 make high-quality mathematics instruction accessible to ALL students, including students with disabilities. This workshop is intended for mathematics teacher educators and researchers. Our overall aims are to illustrate key professional learning strategies employed by Math for All to better equip teachers for planning cognitively demanding mathematics lessons that build on diverse students' strengths and needs, and to share research findings that demonstrate the efficacy of the program. We will demonstrate these strategies by engaging participants in the following activities in a hands-on manner: (1) learning about a neurodevelopmental framework for learning (e.g., Barringer, Pohlman & Robinson, 2010; Pohlman, 2008), (2) applying the neurodevelopmental framework in the analysis of the demands of a mathematical task and a focal student's strengths and needs using segments from a video-recorded third-grade case lesson on multiplication, and (3) reflecting on how instructional strategies for a mathematical task can be adapted to build on the strengths and needs of a focal student without undermining the rigor of the mathematical goals. We will conclude with sharing findings from multiple research studies that have demonstrated the efficacy of the Math for All approach.

### Planned Activities & Working Format & Responsible Person

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| <b>21:30–21:45</b> | <b>Math for All overview &amp; introduction to the neurodevelopmental framework</b><br>Small group and large group discussions (Babette Moeller).   |
| <b>21:45–22:05</b> | <b>Analysis of the demands of the mathematical task of a case lesson</b><br>Viewing of video clip of the case lesson's teacher introducing the math task<br>Hands-on exploration of the math task<br>Small and large group discussion of the demands of the math task (Matt McLeod)   |
| <b>22:05–22:25</b> | <b>Observation of a focal student from the case lesson to understand her strengths and needs</b><br>Viewing of video clip of a focal student working on the math task<br>Small group and large group discussion of the focal students' strengths and needs (Babette Moeller).   |
| <b>22:25–22:45</b> | <b>Discussion of the mathematical goals of the case lesson and instructional strategies employed by the case lesson teacher</b><br>Large group discussion of the mathematical goals of the case lesson's math task<br>Small group viewing of video clips of case lesson teacher working with the focal student and other students as they engage in the math task<br>Large group discussion of instructional strategies and how they align with the focal students' learning profile and support the goals of the lesson (Matt McLeod). |
| <b>22:45–23:00</b> | <b>Discussion of research findings</b><br>Large group discussion (Babette Moeller & Matt McLeod).   |



## WS12: Teaching Mathematics for Social Justice in TVET Colleges

**July 14, 21:30–23:00**

**Location: T132**

**Organiser: Siphokazi Winniefred Vimbelo** (Cape Peninsula University of Technology, South Africa)

### Description:

The organiser is Ms Siphokazi Winniefred Vimbelo from Cape Peninsula University of Technology in South Africa. The workshop will be about teaching mathematics for social justice in TVET colleges. Mathematics is often seen as the most neutral of discipline within a given curriculum, one that is removed from the arguments and controversies of politics and social life (Yeh & Otis, 2019). However, in reality, it is like all other disciplines of learning- it sits within larger fields of social, cultural, and political beliefs and practices (Greenstein & Russo, 2019). It can serve as a powerful means for developing students' understanding of issues of social justice and the students are likely to develop an understanding of both social justice issues and mathematical concepts. One of the greatest challenges in learning to teach mathematics for social justice is a requirement for negotiation and integration of mathematical goals with social justice goals (Yeh & Otis, 2019). Teaching mathematics through social justice becomes a way to make the learning more relevant to students' real – life examples (Harrison, 2015). Hence the workshop on teaching mathematics for social justice. The workshop will consist of 4 components:

- Narratives – participants will be writing narratives about their current praxis and will shared.

- Brainstorming – PowerPoint presentation and Gutstein’s book will be used but we will focus only on his projects.
- Implementation – Participants will be designing lessons based on TMfS and lesson will be presented.
- Reflexive Activity – participants will be reflecting on the workshop and lessons and write narratives.

### Planned Activities & Working Format & Responsible Person

- 21:30–21:50**     **Narratives – about current praxis**  
Templates.
- 21:50–22:10**     **Brainstorming**  
Power Point slides (Gugu’s story and Gutstein’s book (2006) “Reading and writing the world with mathematics”).
- 22:10–22:40**     **Implementation – lesson presentations**  
Gutstein’s projects to be used
- 22:40–23:00**     **Reflexive activity**  
Template

## WS13: Self-made Automata to Teach Mathematics in Preschool

**July 14, 21:30–23:00**

**Location: T222**

**Organiser: Oliver Thiel** (Queen Maud University College, Trondheim, Norway); Piedade Vaz Rebelo (University of Coimbra, Portugal)

### Description:

The workshop disseminates findings from the European research project AutoSTEM. The aim of the project is to investigate how automata can enrich young children’s play to promote a better understanding of Science, Technology, Engineering, and Mathematics (STEM). It aims to provide preschool teachers and other stakeholders of young children’s education of tools and materials to build a didactic path, which is simple, replicable, and valuable in terms of

- 1) promotion of a motivation for STEM, especially mathematics,
- 2) promotion of the development of creative thinking, problem-solving, and comprehension ability, and
- 3) cultural awareness and transversal values such as recycling.

Automata are fascinating mechanical toys. Due to the combination of narrative and mechanical parts, automata have several possibilities for use within education. They are easy to create in the classroom, suitable for the children’s age, with simple to complex designs and motions. In the project, we use a relational play-based pedagogy (Hedges & Cooper, 2018) and a dynamic learning concept (Broström, 2017).

In the workshop, we will present the ‘snapping crocodile’ developed by the project and discuss how they can be used to teach mathematics in the early childhood classroom (preschool and kindergarten). A major part of the workshop will be hands-on work where participants will make their own mechanical crocodile. To participate, you have to prepare: cardboard (at least A4 in size), a wooden skewer or an awl, scissors or a knife, 10 split pins (paper fasteners). Target group for the workshop are preschool teachers and educators.

### Planned Activities & Working Format & Responsible Person

- 21:30–21:50**     **Automata in early childhood mathematics education**  
Short presentation of the project’s findings – O. Thiel
- 21:50–22:40**     **Making a ‘snapping crocodile’**  
Participants make their own crocodile from recycled materials and experience how they are related to maths concepts – O. Thiel
- 22:40–23:00**     **Pedagogical concepts and ideas**  
Participants reflect on and discuss how the presented automata can be used to teach mathematics in preschool – O. Thiel

WS

## WS14: Exploratory Lessons Using Pop-up Cards and Making of Cards

July 14, 21:30–23:00

Location: T226

**Organiser:** Kazumi Yamada (Niigata University, Japan); Takaaki Kihara (Nagaoka Institute of Design, Japan); Anri Yamada (Tama Art University, Japan)

### Description:

(1) Making of cards as the teaching materials of the space figure

We have been continuing workshops on ICME11, ICME12, and ICME13 about teaching spatial figures using pop-up cards.

A static figure is used in the learning of the plane figures. In contrast, it is important to present the shapes of the spatial extent and dynamic movements when a teacher teaches a space figure. When making a card, a three-dimensional card is completed by trial and error, making a cut in a plane (card) plan, and opening and closing a card repeatedly. Especially, the popup card called "the origami architecture" is effective as the teaching material.

(2) Exploratory classes using pop-up cards

Let's create the pop-up card. You will discover that if you make the cards correctly, you can fold the cards and stack. This is a meaningful discovery that easily checks the correctness of the design drawing. While repeating the card making, various questions will arise and discover many properties. For example, they are as follows. "In the blueprint, where are the cut lines? Where are the folds? Are there any secrets to these lines?" In this workshop, we perform a class to discover these mathematical properties for searching with a participant of the part of student.

(3) Making works

Make works with Kihara. Teaching material 1 is a pop-up card that opens at  $90^\circ$ , where the house appears. Teaching material 2 is a pop-up card, where the house appears when it opens at  $180^\circ$ .

### Planned Activities & Working Format & Responsible Person

**21:30–21:50** **Discovery of exploratory mathematical properties through pop-up card creations**

Online Lecture / Kazumi Yamada.

**21:50–22:20** **Exploratory class using pop-up cards**

Online Activity / Kazumi Yamada, Anri Yamada

**22:20–22:50** **Making of pop-up cards**

Online Activity / Takaaki Kihara.

**22:50–23:00** **Question and answer**

## WS15: Challenging Ableist Perspectives on the Teaching of Mathematics: A CAPTeaM Workshop

July 14, 21:30–23:00

Location: T230

**Organiser:** Elena Nardi (University of East Anglia, UK); Irene Biza (University of East Anglia, UK); Solange Hassan Ahmad Ali Fernandez (Universidade Anhanguera de São Paulo, Brazil); Lulu Healy (King's College London, UK); Érika Silos (Universidade Federal Fluminense, Brazil); Angeliki Stylianidou (University of East Anglia, UK)

### Description:

The CAPTeaM project (Challenging Ableist Perspectives on the Teaching of Mathematics) sets out from the assumption that, rather than being the consequence of internal, individual factors, disabled students' oft-reported underperformance in mathematics can result from explicit or implicit exclusion from mathematics learning. The project challenges teaching practices that contribute to such exclusion and that may emanate from ableist perspectives on mathematics. The project's aims cohere with the 2006 United Nations Convention on the Rights of People with Disabilities. The project endorses a Vygotskian historical-cultural perspective and elements of embodied cognition and its data consists of written

responses and video recorded work on two types of tasks, Type I and Type II, by pre- and in-service teachers of mathematics.

In Type I tasks, participants engage with classroom episodes that evidence mathematical contributions which are made by students with a physical disability (e.g. are visually or hearing impaired), have the potential to shift classroom mathematical discourse towards creatively unexpected turns and may bring learning benefits to all in class. Said episodes are selected from the databases of the Brazil-based (Rumo à Educação Matemática Inclusiva) and UK-based project partners. In Type II tasks, participants engage in small groups with solving a mathematical problem while at least one of them is temporarily and artificially deprived of access to a sensory field or familiar channel of communication. Work on tasks concludes with sharing reflections in plenary discussion.

This workshop will engage participants with tasks of Type I and II and will last 90 minutes.

### Planned Activities & Working Format & Responsible Person

- |                    |  |
|--------------------|--|
| <b>21:30–21:35</b> | <b>What is CAPTeaM?</b><br>The project lead presents a brief introduction to the CAPTeaM project. / Nardi, brief exposition  |
| <b>21:35–21:55</b> | <b>A CAPTeaM activity (Type I)</b><br>Participants engage with a CAPTeaM Type I activity. / Nardi, Biza; participants work in small groups   |
| <b>21:55–22:10</b> | <b>Plenary discussion I</b><br>Participants share the work generated in small groups with the whole group and reflect on the experience. / Nardi, Biza; whole group discussion   |
| <b>22:10–22:30</b> | <b>A CAPTeaM activity (Type II)</b><br>Participants engage with a CAPTeaM Type II activity. / Nardi, Biza; participants work in groups of three  |
| <b>22:30–22:45</b> | <b>Plenary discussion II</b><br>Participants share the work generated in the groups with the whole group and reflect on the experience. / Nardi, Biza; whole group discussion  |
| <b>22:45–23:00</b> | <b>CAPTeaM findings: the present and the future, Q&amp;A, reflections and evaluation</b><br>The project lead outlines project findings so far and maps plans for the future. Participants ask questions and reflect on/evaluate the experience of participating in the workshop. / Nardi, brief exposition; then, all. |

WS

## WS16: Networking Design Approaches: Around the Teaching of Mathematical Proof

**July 14, 21:30–23:00**      **Location: T234**

**Organiser:** Tatsuya Mizoguchi (Tottori University, Japan); Ignasi Florensa (Escola Universitària Salesiana de Sarrià, Spain); Koji Otaki (Hokkaido University of Education, Japan); Hiroaki Hamanaka (Hyogo University of Teacher Education, Japan)

### Description:

This workshop is based on part of an ongoing research project regarding the cultural and anthropological study on the development of competencies of mathematical proof throughout of secondary school. This workshop will focus on the design of teaching, especially for mathematical proof task. For this, various designs based on different theoretical approaches will be compared and their characteristics will be considered. Therefore, the key questions of this workshop are as follows: (1) How teaching of mathematical proof can be designed with each approach; (2) What characteristics each approach has in the design process; (3) How does each approach complement the others?

In this workshop, we will consider three different approaches: Study and research paths (Q-A map) in the Anthropological Theory of the Didactic; Japanese problem-solving lesson model (so called open approach); Substantial Learning Environment. Based on the planned structure shown below, this workshop accesses the above questions by designing teaching for a common mathematical task and then comparing (networking) them. There will be cultural and theoretical differences in teaching design. In this workshop,

we will inquire the possibility of new approaches to these problems through collaboration with participants.

The workshop will be organized jointly by Japan and Spain. In addition, the workshop will be conducted in cooperation with the following prospective contributors (project members): Yoshitaka Abe (Niigata, University), Terumasa Ishii (Kyoto University), Hiroyuki Kumakura (Shizuoka University), Susumu Kunimune (Shizuoka University), Takeshi Miyakawa (Waseda University), Yusuke Shinno (Hiroshima University), Yuki Sugimoto (Nagasaki University).

### **Planned Activities & Working Format & Responsible Person**

- 21:30–21:35**     **Introduction and overview of WSG**  
All participants / T. Mizoguchi.
- 21:35–21:50**     **Introducing the common task**  
All participants / H. Hamanaka
- 21:50–22:20**     **Short keynotes: theoretical tools and the teaching-designs**  
All participants / I. Florensa, T. Mizoguchi, & K. Otaki
- 22:20–22:50**     **Discussing along with the key questions**  
All participants / I. Florensa, T. Mizoguchi, K. Otaki, & H. Hamanaka
- 22:50–23:00**     **Summarizing: reflections and further considerations**  
All participants / I. Florensa, T. Mizoguchi, K. Otaki, & H. Hamanaka

## **WS17: Linguistic and Logical Methodological Tools to Address Language Diversity in Mathematics Education**

**July 14, 21:30–23:00**                      **Location: T302**

**Organiser: Viviane Durand-Guerrier** (University of Montpellier, France); **Cris Edmonds-Wathen** (Charles Darwin University, Australia); **Faiza Chellougui** (University of Carthage, Tunisia); **Judith Njamgong Ngonsap** (University of Yaoundé, Cameroun); **Jean-Jacques Salone** (CUFR of Mayotte, France)

### **Description:**

The aim of the workshop is to share with an international audience the linguistic and methodological tools we are developing in our own multilingual contexts in order to discuss the possibility of their generalisation, how to improve them for wider use and to initiate international collaborations involving a variety of languages.

The main idea underlying this proposal is that in multilingual contexts differing grammatical structures of languages might affect the process of teaching and learning mathematics, whatever the level of instruction, considering that switching from one language to another in a classroom might be both an obstacle or a resource, and that translating even the most straightforward of mathematical statements from a language to another presents challenges. We also consider the issue raised by the translation of transcripts of classroom situations for communication of our research for an international audience this being even more accurate for research on language diversity.

The workshop will comprise one session of 90 minutes, with previous asynchronous activities consisting in watching videos presenting our methodological tools, answering a short questionnaire, including a short transcript to translate in the participants own preferred language. During the session, the participants will 1/ Share questions and comments on the methodological tools shared via the videos; 2/ Small groups discussions on grammatical issues raised by the translation of the transcripts, and the relevance and the limits of the shared methodological tools; 3/ Collective synthesis of the small groups work; 4/ Perspective for possible future collaborations.

### **Planned Activities & Working Format & Responsible Person**

- 21:30–21:45**     **Sharing questions and comments on the methodological tools**  
Collective discussion / VDG & CW
- 21:45–22:45**     **Working and discussing on translation issues in multilingual contexts and in communication of research results**  
Small groups works on issues arising while the translation / FC, VDG, CEW, JNN, JJS

Report of small groups works and collective discussion / FC, VDG, CEW, JNN, JJS

**22:45–23:00**

**Toward future collaborations**

Collective discussion and paths for going on working on this topic / VDG & CEW

## **WS18: Topological Approach to Game Theory**

**July 14, 21:30–23:00**

**Location: T305**

**Organiser: Giovanna Bimonte** (University of Salerno, Italy); **Francesco Saverio Tortoriello** (University of Salerno, Italy); **Ilaria Veronesi** (University of Salerno, Italy)

### **Description:**

We present a laboratory developed in the mathematics activities during the lessons of the research –project “Mathematical High School” at the University of Salerno.

In Italian higher education, the topic of “Game Theory” is not included in the ministerial indications

of the mathematics curriculum. Students do not have the prerequisites that allow them to understand and solve optimisation problems of several variables. We decided to use a geometric approach applied to localisation problems. Competitive Localisation Models are concerned with the fact that some structure is already in the market and the new structure will compete for market share (Hotelling1929).

We consider a continuous location optimization problem, where an optimal location is found in a continuum on a plane. We introduce the Voronoi diagram in order to solve the location problem, in which the number of players is determined exogenously. We use Delaunay triangulation to find the equilibrium point, and consider some generalizations of the of the ordinary Voronoi diagram.

The solution of the problem in the planar case with Euclidean distances and a variety of functions of attraction leads to a finite polynomial algorithm in the number of consumers. Using a dynamic geometry software we construct our case study on the Cartesian plane, we check how the results change as the starting conditions vary and we obtain the solutions without even performing the simple calculations required by the Cartesian geometry to find the equilibrium point.

### **Planned Activities & Working Format & Responsible Person**

**21:30–21:40**

**State of art**

Group activities – G. Bimonte.

**21:40–21:50**

**Dynamic mathematics software**

Group activities – F.S. Tortoriello.

**21:50–22:00**

**Topological approach to Game Theory**

Group activities – I. Veronesi

**22:00–22:15**

**Laboratory of positional games**

Group activities – G. Bimonte, F.S. Tortoriello, I. Veronesi.

**22:15–23:00**

**Laboratory on the topological approach, solutions and discussions**

Group activities – G. Bimonte, F.S. Tortoriello, I. Veronesi.

## **WS19: Global Math Stories: Travel the World, Explore Social Justice, and Deepen Your Understanding of Math**

**July 14, 21:30–23:00**

**Location: T306**

**Organiser: Chadd W. McGlone** (Teachers2Teachers Global, United States); **Hanna Haydar** (City University New York - Brooklyn College, United States); **Paola Castillo** (Teachers2Teachers Global, Ecuador)

### **Description:**

Mathematics class comes alive through global connections. Unfortunately, many classroom teachers lack the time and resources to require to make these connections. GlobalMathStories.com is a free resource for making global connections in the classroom. The site consists of approximately 75, one-page stories written by people from around the World. Teachers may choose to write their own lessons from the stories

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or use one of the lessons already developed. Each story contains resources to further explore the culture and social justice questions behind the mathematics.

In this workshop, participants will learn about the site and how to use it in their classroom. First, they will participate in a mathematical task based on one story. In addition to completing the tasks, individuals will consider social justice questions associated with the story. Throughout, the presenters will emphasize the importance of reading an audience and telling a great story during the lesson. Next, participants will explore a second story. Working as teams, groups will propose mathematical tasks they might create based on the story. Additionally, they will propose social justice questions that arise from their exploration. Next, participants will choose a story from the site to develop. They will present the story to the audience and describe how a lesson would progress, complete with a social justice question.

Finally, everyone will discuss how connections of local and global cultures enhance student learning. They will be invited to contribute stories to the site and to share it with educators and authors in their communities.

### **Planned Activities & Working Format & Responsible Person**

**21:30–21:50** **Model lessons drawing from a Global Math Story (GMS). GMS set the reader in a culture and provide opportunities to complete mathematical tasks based on that story. The presenter will outline how a lesson might look using a GMS.**

Cooperative learning in groups. Chadd McGlone Hanna Haydar, and Paoula Castillo

**21:50–22:10** **The presenter will tell another GMS and allow participants to work in groups to design a mathematical task based on that story. An emphasis will be on helping participants understand they must bring students into the story. Additionally, participants will learn how easy it is to connect stories to mathematical tasks.**

Cooperative learning in groups. Chadd McGlone Hanna Haydar, and Paoula Castillo

**22:10–22:40** **Participants will choose one of 60 available stories from which to develop a lesson. They will create a mathematical task from the story and present the story to others in the workshop.**

Cooperative learning in groups. Chadd McGlone Hanna Haydar, and Paoula Castillo



## **WS20: Enhancing Oral Practice in Mathematics Class**

**July 14, 21:30–23:00**

**Location: T309**

**Organiser: Luca Agostino** (Université d'Evry, France)

### **Description:**

Presentation of the speaker: Luca Agostino:

Mathematics teacher in the secondary school, I work also as teacher trainer and prepare and follow the entering in the school system for young teacher. My research activity is developed in Université d'Evry and it is focused on the oral practices in the maths classes and on the international comparisons of the teacher training systems.

This workshop will allow participants to share our experience of testing out the construction of mathematical reasoning by giving the students enough time to build complex argumentative sequences. In the french high school La Plaine de Neauphle we use whiteboards to induce group work, enhance cooperative and oral skills though students debate.

Starting and ending rituals: starting and endings of a class time are often the right time to bring the students to talk: whether it is to recap the last session or sum up this one, to correct homework or to answer a few questions in order to jog one's memory about last chapter, it can be a useful time if well managed.

Running oral examinations instead of written ones will grow the students accustomed to talk in front of a crowd (or a at least a jury). The students being both speaker and listener, they will integrate the right attitude and rules of oral presentations, with the help of the teacher, who can present a first set of rules, which will be sharpened during the course of the year and the student's suggestions.

### **Planned Activities & Working Format & Responsible Person**

**21:30–21:40**     **Starting and ending rituals**  
Presentation, exchanges and debate

**21:40–21:50**     **Oral examinations**  
Presentation, exchanges and debate.

**21:50–22:50**     **Educational walls**  
Group work and roleplay

## **WS21: Mathematical Performance-based Learning Workshop**

**July 14, 21:30–23:00**                      **Location: T323**

**Organiser: Jing Yang**, Fan Zou, Shuyang Sun (Math Teacher Research Group from Yungu School, China); Xuanzhe Sun, Chuan Qu, Chengzhi Yu (Students from Hangzhou Yungu School, China)

### **Description:**

We are a group of teachers from Hangzhou Yungu School. In this workshop, we will share how to build a mathematical performance task of real-life situation, by going through some daily practices we had in the past four years. We believe that by real-world performance-based learning, students are guided to apply mathematical knowledge and skills both inside and beyond subject learning, to help students to perceive math as sensible and worthwhile. Not only content knowledge and procedural fluence skills can be improved, but also problem solving and mathematical reasoning skills. We want to inspire students to recognize and apply connections among math and outside of math.

### **Planned Activities & Working Format & Responsible Person**

**21:30–21:40**     **Warm-up:**

**(1) Self-introduction**

**(2) Icebreaking time: a math board game created by Yungu students**

**(3) Clarify the major goals of the workshop**

Material: Board games designed by Yungu students / Portable Tables and Chairs

Working Format: Presentation and a math game

Responsible Persons: 3 Yungu students and 3 teachers together.

**21:40–22:10**     **3 Presentations by Yungu students**

Material: Projector (Apple TV preferred)

Working Format: Presentations

Responsible Persons: 3 Yungu students and Jing Yang

**22:10–22:50**     **Daily practices of mathematical performance-based learning in Yungu School**

Material: Projector (Apple TV preferred)

Working Format: Presentations

Responsible Persons: Fan Zou/Shuyang Sun

**22:50–23:00**     **Summary**

Working Format: Presentation

Responsible Person: Jing Yang

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## WS22: Exploring the Role of Online Interactive Technology in Supporting Dialogue in Mathematics Classrooms: Lesson Study in a Chinese Primary School

**July 14, 21:30–23:00** Location: T319

**Organiser:** Qian Liu<sup>1</sup>, Yuan Zhang<sup>2</sup>, Manqi Yu<sup>2</sup>, Xianzhong Chen<sup>2</sup>, Pan Liu<sup>2</sup>, Taifeng Shen<sup>2</sup>, Huayan Sun<sup>2</sup>, Yingying Zhang<sup>2</sup>, Yanyu Zhang<sup>2</sup>, Guijuan Wang<sup>2</sup> (<sup>1</sup>University of Cambridge, China; <sup>2</sup>Hangzhou Yungu School, China)

### Description:

The theory-informed and practice-based workshop is organised and presented by the mathematics research group in Hangzhou Yungu School directed by Yuan Zhang and the PhD candidate, Qian Liu. The knowledge, experience and teaching practice shared and discussed here were derived from the two-year lesson study program conducted in Hangzhou Yungu school. The lesson study inquiry was underpinned by the theoretical assumption that the pedagogically appropriate use of digital technologies could resource, expand and deepen classroom dialogue and in turn promote students' mathematical attainment, understanding and higher order thinking (e.g. Howe et al., 2019; Mercer & Sams, 2007). Based on this, ten primary mathematics teachers and the researcher explored and investigated what affordances of online interactive technology are potentially beneficial for primary mathematics dialogue and how the affordances can be effectively enacted with dialogic teaching to support productive classroom dialogue. Our workshop, bridging dialogic theory and school-based classroom practices, aims to enrich participants' understanding about the role of online interactive technology in mathematics teaching and learning from the dialogic perspective. Secondly, we would like to share our developed teaching models respectively applied in the whole-class and student-led group sessions, the corresponding pedagogical approaches and strategies exemplified by real lesson cases. More importantly, the workshop designed and organised in a dialogic manner aims to draw participants into a diverse dialogue. We will invite participants to share their comments, suggestions and questions and design together the potential pedagogical use of online interactive technologies for productive dialogue based on one provided lesson episode.

### Planned Activities & Working Format & Responsible Person

**21:30–21:35** **Opening and welcoming & Clarifying main objectives and structure of the workshop & Ice-breaking online interactive activity**

Introduction and online interaction (Pan Liu)

**21:35–21:50** **Overview of the theoretical framework and the lesson study inquiry in Yungu school & The potentially supportive role of online interactive technology in classroom dialogue**

Presentation (Qian Liu) Discussion (Qian Liu)

**21:50–22:15** **Lesson cases**

Presentation (Xianzhong Chen & Manqi Yu)

**22:15–22:45** **Instructional design employing digital technology**

Group activity & Discussion (Yuan Zhang)

**22:45–23:00** **Comments and additional questions & Conclusion**

Q & A (Yuan Zhang, Qian Liu)

## WS23: Frame Thinking in Adaptive Learning

**July 14, 21:30–23:00** Location: T223

**Organiser:** Yang Cao (Nanzhong Education, China)

### Description:

Name of organizer: Yang Cao

- Former product manager of iFLYTEK ZHIXUE Adaptive Learning System
- Former assistant of iFLYTEK's Vice-President
- Sole author of series of Learn Math, This Way!, published by University of Science and Technology of China Press(2019/2020), ISBN: 978-7-312-04688-9/978-7-312-05014-5

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This WSG aims at how to apply frame thinking in automatic learning controlled by students themselves.

Followed the opinion described in Learn Math, This Way!, Yang Cao will give a brief introduction to Frame Thinking and some examples about alleviating the burdens of K12 student's maths learning and enhancing their abilities on mathematical modelling.

Then, other participants, including some experts in adaptive learning, will illustrate their practices in structured maths problems solving, supported by abundant and vivid data collected from middle and primary schools.

Finally, all the staff will discuss how to improve the efficiency of maths learning empowered by innovation in mathematical thinking method and by IT product optimization.

### Planned Activities & Working Format & Responsible Person

- 21:30–21:50**     **How frame thinking makes maths easy and interesting?**  
PPT, sponsored by Yang Cao
- 21:50–22:30**     **Adaptive learning practices put into execution in chinese middle and primary schools**  
PPT, sponsored by other participants
- 22:30–23:00**     **Free discussion (innovation in mathematical thinking method or IT product optimization?)**  
Not defined, responsible person: Yang Cao

## WS24: Mathematics Learning and Mathematics Games

**July 14, 21:30–23:00**     **Location: T219**

**Organiser: Hongliang Shi** (No. 2 High School of East China Normal University, China); Fanglin Tian (No. 2 High School of East China Normal University, China); Zhiyu He (No. 2 High School of East China Normal University, China)

### Description:

The organizers in the workshop are math teachers instructing students aged 12-18. Many of the teachers are members of a program named Mathematics Learning and Mathematics Games. Professor Shi is the leader of the organizers. He is the deputy director of Basic Education and Lifelong Education Development Department of East China Normal University, the vice principal of No.2 High School of East China Normal University, and the host of the Mathematics Base of the Fourth Phase of Shanghai Famous Principals and Educators Project and the council member of Shanghai Mathematics Society.

The aim of this workshop is to show our work on combining mathematics learning and mathematics games and to hold a mathematics festival.

In the workshop, we will share the detail of our practice, give a brief review of the results to the questionnaires and interviews, and share our experiences about the series of activities. What's more, we will hold a mathematics festival, which includes 15 games. Every participant in our workshop is allowed to play the games they are interested in and can switch to another game as they wish. Different games are played at different tables at the same time and players can discuss the answers with table leaders. In particular, we will display several traditional Chinese games such as Chinese Rings, Tangram, Magic Square and so on. Welcome game lovers and math fans from all over the world to join us.

### Planned Activities & Working Format & Responsible Person

- 21:30–21:40**     **Aims and meanings**  
Lecture / Hongliang Shi
- 21:40–22:00**     **What have we done?**  
Lecture / Fanglin Tian
- 22:00–22:50**     **Mathematics festival**  
Games / Hongliang Shi, Fanglin Tian, Zhiyu He
- 22:50–23:00**     **Conclusions**  
Lecture / Zhiyu He

## WS25: Mentoring Student in the Innovative Mathematics Mini Projects

July 14, 21:30–23:00

Location: T225

**Organiser:** Jieliang Wang (High School Affiliated to Fudan University, China); Xiao Enli (High School Affiliated to Fudan University, China); Yang Liting (High School Affiliated to Fudan University, China); Guo Tianxiang (Shanghai Caoyang No. 2 High School, China)

### Description:

This theme has created a training model for talent education. Through this model, the educational and teaching notion of educating people have been set, and the basic principles for guiding students in the innovative mathematics mini projects have been determined. Those principles are: principle of basis, principle of self-motivation, principle of capacity, principle of innovation, principle of rigor. In order to achieve the expected goals and the basic principles, the specific practices of our team are as follows:

1. Strengthen the cultivation of thinking ability.
2. Offer 6 series of mathematics lectures.
3. Organize seminars to cultivate students' important abilities and habits.
4. Specific methods to instruct students to conduct innovative mathematics mini projects.
5. Instruct students to participate in extra-curricular activities.
6. Promotions of innovative mathematics mini projects.

We aim to enhance the academic ability of the students by implementing the research and learning model based on writing innovative mathematics essays in the high school stage. By instructing students to participate in various scientific and technological innovation activities, mathematics competition and academic exchange activities at home and abroad, we help students to dedicate to their own research in mathematics from basic mathematic problem study. We have concluded a process for guiding students to conduct innovative mathematics mini projects and have been following up with the growth of their academic abilities in the long run.

### Planned Activities & Working Format & Responsible Person

<b>21:30–21:37</b>	<b>Introduction of team leader and team members, the background and expertise of the team leader</b> Presentation / Jieliang Wang, Xiao Enli, Yang Liting, Guo Tianxiang
<b>21:37–21:55</b>	<b>Detailed description of the topic theme</b> Presentation / Jieliang Wang, Guo Tianxiang
<b>21:55–22:07</b>	<b>Specific practices of our team</b> Presentation / Jieliang Wang, Guo Tianxiang
<b>22:07–22:25</b>	<b>Specific methods to instruct students to conduct innovative mathematics mini projects</b> Presentation / Jieliang Wang, Guo Tianxiang
<b>22:25–22:50</b>	<b>Instructing students to participate in extra-curricular activities and promotions of innovative mathematics mini projects</b> Presentation / Jieliang Wang, Guo Tianxiang
<b>22:50–23:00</b>	<b>Interactive communication</b> Jieliang Wang, Xiao Enli, Yang Liting, Guo Tianxiang

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## WS26: Folding for Fractional Understanding

July 14, 21:30–23:00

Location: T313

Organiser: Björg Jóhannsdóttir (California State University, Stanislaus); Heather Coughlin (California State University, Stanislaus)

### Description:

This interactive online workshop is run by Dr. Heather Coughlin and Dr. Björg Jóhannsdóttir, veteran teacher educators and designers of professional development workshops for teachers in mathematics.

The goal of this workshop is to introduce paper strips as manipulatives to foster understanding of fractions. Attendees gain appreciation for the versatility of the paper strip to visualize concepts, link fractions to the whole numbers, and building arithmetic algorithms. By manipulating a paper strip, fraction definition emerges, and addition, subtraction, multiplication, and division of fractions actually make sense, as the operations become alive in the students' hands. Each activity presented in the workshop can be used, as exhibited, in the classroom, so attendees walk away with creative activities to build students' understanding of fractions.

Grade Level: 3-5.

Required Material: Paper Strips (approx. 2 cm), writing utensils.

### Planned Activities & Working Format & Responsible Person

21:30–21:32	<b>Introduction</b> Dr. Coughlin and Dr. Jóhannsdóttir
21:32–21:37	<b>Definition of fractions</b> Dr. Jóhannsdóttir
21:37–21:42	<b>The paper strip as a number line</b> Dr Coughlin
21:42–22:53	<b>Group work: addition, multiplication, and division of fractions with paper strips</b> Dr. Coughlin and Dr. Jóhannsdóttir
22:53–23:00	<b>Thank you and questions</b> Dr. Coughlin and Dr. Jóhannsdóttir

## WS27: Simulation Games for Geometry Learning and the Development of Mathematical Language

July 14, 21:30–23:00

Location: T202

Organiser: Angela Piu (University of Valle d'Aosta, Italy); Cesare Fregola (University of Molise, Italy)

### Description:

The aim of the workshop is to present educational interventions based on collaborative simulation games targeting geometry learning among primary school pupils. These games have been conceptualized, developed, and tested as part of a research project called Simulandia. They are designed to advance students' understanding of key geometrical concepts and to stimulate the development of related skills and competences, including the gradual appropriation of mathematical language. Their structure and characteristics foster the guided discovery of geometry concepts, eliciting internal and external representation of these concepts, the sharing and communication of related meanings, and systematic review and formalization of new learning at the end of the game.

Participants will be introduced to Cartolandia, a simulation game on isometries. They will first be familiarized with the features of the game and the underlying theoretical and methodological framework, which informed the development of all the Simulandia games. Next, they will reflect on and discuss how the teaching/learning process is related to the structure and unfolding of the game. The aim is to provide attendees with an experiential appreciation of how children can construct mathematical language starting from everyday language, and how they can abstract – both spontaneously and in response to the demands of the game – the concepts they progressively discover, representing them in different semiotic registers.

Finally, participants will debate the transferability of the simulation game technique to the settings where they teach, and discuss potential new lines of inquiry.

### **Planned Activities & Working Format & Responsible Person**

- 21:30–21:40**    **Introduction**  
Presentation and training agreement / Piu
- 21:40–21:55**    **Theoretical-methodological framework**  
Presentation and explanation of key concepts underpinning the design of the simulation games - Fregola
- 21:55–22:25**    **Experiential demonstration**  
Observation of the teaching-learning process based on video clips / Piu - Fregola.
- 22:25–22:45**    **Debriefing**  
Analysis of observation outcomes / Piu - Fregola
- 22:45–22:55**    **Exploration of follow-up lines of inquiry**  
Free discussion / Piu
- 22:55–23:00**    **Conclusions**  
Piu - Fregola

