THE TRANSITION FROM MATHEMATICAL ARGUMENTATION TO MATHEMATICAL PROOF, A LEARNING AND TEACHING CHALLENGE

Nicolas Balacheff

Univ. Grenoble Alpes, France

Mathematical proof is the backbone of mathematics as a scientific discipline. All along the 20th century, the meagre success of its teaching prompted most of the decision makers to postpone it until children have achieved a certain cognitive development. Research outcomes of the last decades suggest that the teaching challenge can be overcome, hence the nowadays wide consensus that mathematical proof ought to be part of curricula at whatever grade from kindergarten to university. To properly express this objective requires finding an adequate characterization of proof and the right words while one has been accustomed to using several different ones as mere synonymous.

First, I suggest to slightly change the didactical problem from learning proof to understanding how can be asserted the truth value of a statement in mathematics at different grades. This requires to tighten the links between problem-solving and proving, as well as between knowing and proving. I develop this position focusing on three terms: control, argumentation and proof. The choice of these terms intends to denote three regimes of validation whose respective weights change along the continuum from solving a problem to communicating its solution according to the mathematics standards in force at a given grade. Second, I shall shape the relations between argumentation and proof from an epistemological and didactical perspective. Doing this, I will pay attention to our linguistic, cultural and epistemological differences.

Although the historical roots of mathematical proof could give it legitimacy, the concept of mathematical argumentation will be a didactic concept and not the transposition of a mathematical one. The inherent social nature of argumentation would otherwise make a lasting impact on the understanding of mathematical proof. Although being the product of a human activity which certification is the outcome of a social process, a mathematical proof is independent of a particular person or group. The standardization of proof in mathematics, in addition to the institutional character of its theoretical reference, entails its depersonalization, decontextualization and atemporality. While argumentation is intrinsically dependent on an agent, individual or collective, and is "situated".

Eventually, the characteristics of mathematical argumentation must not only distinguish it from other types of argumentation in order to manage its evolution to mathematical norms, but it must also be operational when it comes to arbitrating students' proposals in order to organize and capitalize on them in the classroom knowledge base. How, for example, can be arbitrated the case of the generic example that balances the general and the particular; a balance found at the end of a contradictory debate seeking an agreement which should be as little as possible a compromise?